Lecture 18: B-physics and the Unitarity Triangle

Nov 1, 2016

Some slides taken from lectures given by Marcella Bona
at University College, London

Review from Last Time: CP Violation and the CKM Matrix

- CP Violation first observed in Kaon system in 1964
  - Because Kaon mass low, only 3 observables
    - $|\eta_{+-}| \equiv \frac{A(K_L \to \pi^+ \pi^-)}{A(K_S \to \pi^+ \pi^-)}$
    - $|\eta_{00}| \equiv \frac{A(K_L \to \pi^0 \pi^0)}{A(K_S \to \pi^0 \pi^0)}$
    - $\delta_\ell = \frac{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) - \Gamma(K_L \to \pi^+ \ell^- \overline{\nu}_\ell)}{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) + \Gamma(K_L \to \pi^+ \ell^- \overline{\nu}_\ell)}$
- One source of CP violation: the CKM Matrix
  - In Wolfenstein parameterization:

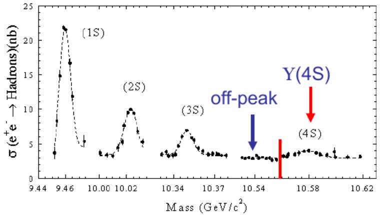
$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Is this the *only* source of CP Violation?
- To answer this question, need to make many measurements and check if all consistent with coming from same choice of CKM parameters
- B system provides rich laboratory for doing this

Sources of B-hadrons

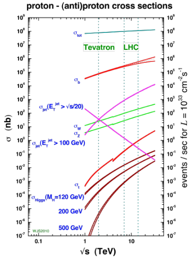
- CP violating effects small
  - Need large number of B mesons to study decay rates with high accuracy
- Two strategies:

$$e^+e^- \to \Upsilon(4s) \to B\overline{B}$$



- Just above threshold
- Only B^+ and B^0
- Coherent stats with no additional particles

$$pp \text{ or } p\overline{p} \to b\overline{b} + X$$

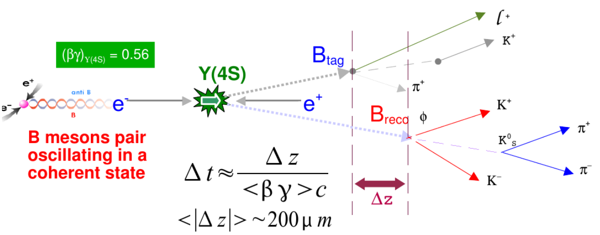


- Very large cross section, but less friendly environment
- Allows studies of B_s and B baryons, was well as $B^\pm$ amd B^0

$e^+e^- \to \Upsilon(4s)$: How do the $B\overline{B}$ pairs behave?

- B and $\overline{B}$ come from $\Upsilon(4s)$ in a coherent $L = 1$ state
  - $\Upsilon(4s)$ is $J^{PC} = 1^{--}$
  - B mesons are scalars
  - Thus, $L = 1$
- $\Upsilon(4s)$ decays strongly so B and $\overline{B}$ produced as flavor eigenstates
  - After production, each meson oscillates in time, but *in phase* so that at any time there is only one B and one $\overline{B}$ until one particle decays
  - Once one B decays, the other contines to oscillate, but coherence is broken
  - Possible to have events with two B or two $\overline{B}$ decays
- This common evolution will become important for CP studies
  - Time integrate asymmetries vanish for cases where CP violation comes from mixing diagrams
  - More on this later
- In addition, in center-of-mass, B hadrons have almost no momentum
  - Difficult to distinguish which tracks come from B and which from $\overline{B}$

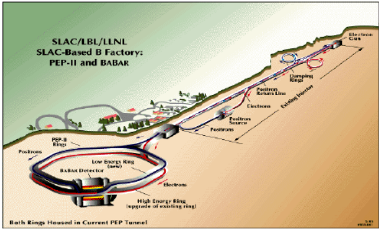
Asymmetric B-Factories

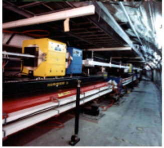


- e^+ amd e^- beams with different energies
  - $\Upsilon(4s)$ boosted along beamline
  - B mesons travel finite distance before decaying
  - Typical distance between decay of the two B mesons: $\sim 200\ \mu$m
- Two B-factories built:
  - SLAC
  - KEK

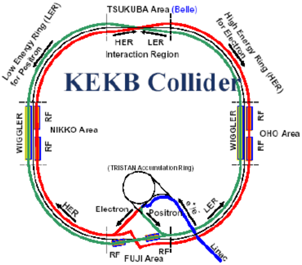
PEP-II and KEKB

PEP-II
- 9 GeV e^- on 3.1 GeV e^+
- $\Upsilon(4S)$ boost: $\beta\gamma = 0.56$

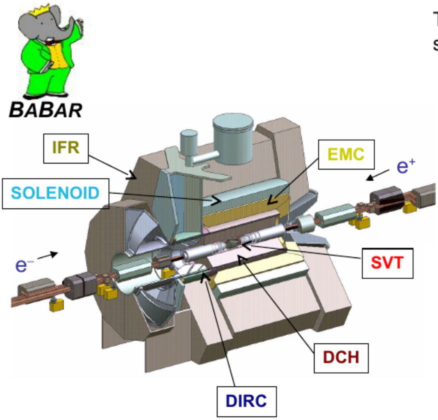




KEKB
- 8 GeV e^- on 3.5 GeV e^+
- $\Upsilon(4S)$ boost: $\beta\gamma = 0.425$

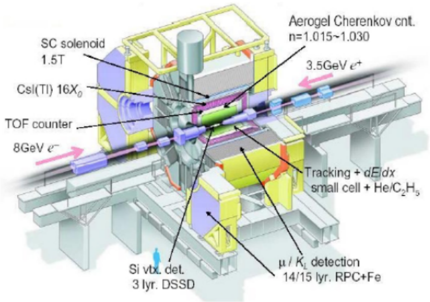


BABAR and Belle



The differences between the two detectors are small. Both have:

- **Asymmetric design.**
- **Central tracking system**
- **Particle Identification System**
- **Electromagnetic Calorimeter**
- **Solenoid Magnet**
- **Muon/K^0_L Detection System**
- **High operation efficiency**



Back to the CKM Matrix

- Reminder:

$$
\begin{aligned}
V_{CKM} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{ds} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\
&\approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}
\end{aligned}
$$

- Note, from the explicit form, you can prove:

$$\rho + i\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

- Unitarity insures $VV^\dagger = V^\dagger V = 1$. Thus

$$
\begin{aligned}
\sum_i V_{ij}V_{ik}^* &= \delta_{jk} \text{ column orthogonality} \\
\sum_j V_{ij}V_{kj}^* &= \delta_{ik} \text{ row orthogonality}
\end{aligned}
$$

- Eg:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

The Unitarity Triangle

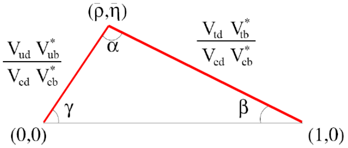
- From previous page

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

- Divide by $|V_{cd}^* V_{cb}|$:

$$\frac{V_{ud}V_{ub}^*}{|V_{cd}^* V_{cb}|} - 1 + \frac{V_{td}V_{tb}^*}{|V_{cd}^* V_{cb}|} = 0$$

- Think of this as a vector equation in the complex plane
- Orient so that base is along x-axis

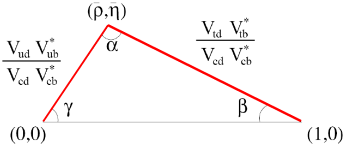


- Reminder from previous page:

$$\rho + i\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

The Measurement Game Plan

- Want to test if matrix is unitary
  - Failure of unitarity means new physics
- Make *many* measurements of sides and angles to over-constrain the triange and test that it closes

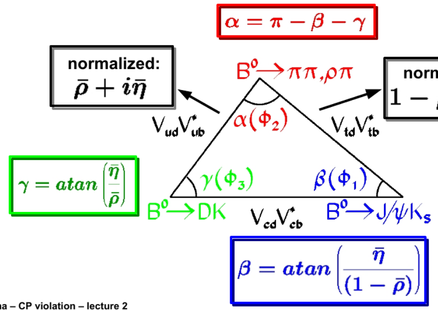


$$\begin{aligned}
\alpha &\equiv arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] \\
\beta &\equiv arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*] \\
\gamma &\equiv arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]
\end{aligned}$$

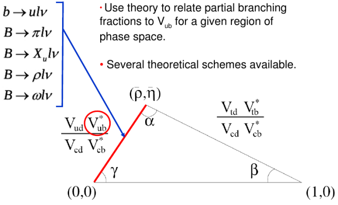
Examples:

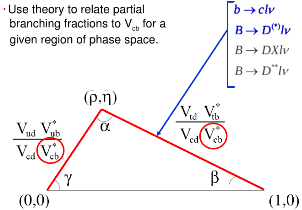
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

many observables
functions of $\bar{\rho}$ and $\bar{\eta}$:
overconstraining



Measuring the Sides (example): Semileptonic Decays

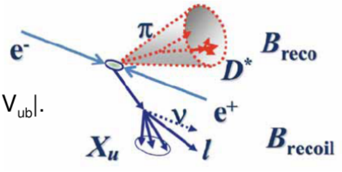




- Requires precise measurement of branching fractions
- Must correct for fact that b-quark is bound in a meson

Side measurements: V_{cb} and V_{ub}

- $|V_{ub}| \propto BR(B \to X_u l\nu)$ in a limited region of phase space.
- similarly for $|V_{cb}|$ $$\frac{d\Gamma(\bar{B} \to D^* l^- \bar{\nu})}{d\omega\, d\cos\theta_l\, d\cos\theta_V\, d\chi} \propto F^2(\omega, \theta_l, \theta_V, \chi) |V_{cb}|^2$$
  - F is a form factor.
  - Need theoretical input to relate the differential rate measurement to $|V_{cb}|$.
- Reconstruct both B mesons in an event.
  - Study the B_{recoil} to measure V_{ub}.
  - Measure BR as a function of $$q^2_{l\nu},\ m_X,\ m_{MISS} \text{ or } E_l$$ and use theory to convert these results into $|V_{ub}|$.



- Can study modes exclusively or inclusively.
- Several models available to estimate $|V_{ub}|$ and $|V_{cb}|$
  - The resulting values have a significant model uncertainty.

Angle Measurements: Types of CP Violation

- Three different categories
  - Direct CP Violation

$$Prob(B \to f) \neq Prob(\overline{B} \to \overline{f})$$

  - Indirect CP Vioation (CPV in mixing)

$$Prob(B \to \overline{B}) \neq Prob(\overline{B} \to B)$$

  - CP Violation between mixing and decay
- Third category cleanest theoretically since no issues of final state interations
- Always need more than one amplitude to allow interference

CP Violation and Phases

- CP conserved:

$$\mathcal{H} = \sum_j \mathcal{H}_j + \sum_j \mathcal{H}_j^\dagger$$

  where $CP\mathcal{H}_jCP = \mathcal{H}_j^\dagger$.

- CP violated:

$$\mathcal{H} = \sum_j e^{i\phi_j}\mathcal{H}_j + \sum_j e^{-i\phi_j}\mathcal{H}_j^\dagger$$

  where each piece acquires its phase from a particular combination of CKM matrix elements. The result then is that while $CP\mathcal{H}_jCP = \mathcal{H}_j^\dagger$, in general, $CP\mathcal{H}CP \neq \mathcal{H}$.

Simplest Case for CP Violation

- If one single part $\mathcal{H}_j$ of the weak Hamiltonian is responsible for the decay $B^0 \to f$ then

$$\begin{aligned}\langle f|\mathcal{H}|B^0\rangle &= \langle f|e^{i\phi_j}\mathcal{H}_j|B^0\rangle &&= \langle f|e^{i\phi_j}CP\mathcal{H}_j^\dagger CP|B^0\rangle \\ &= \eta_f e^{2i\phi_j}\langle f|e^{-i\phi_j}\mathcal{H}_j^\dagger|\overline{B}^0\rangle &&= \eta_f e^{2i\phi_j}\langle f|\mathcal{H}|\overline{B}^0\rangle,\end{aligned}$$

  where η_f is the value of CP for the state f.

- Interference in the decay of a neutral B depends on the weak phases ϕ_j, which come from the CKM matrix, and on the phase introduced by M_{12}.

- Mixing results from box diagram. For M_{12} itself, the dominant diagram has t-quark intermediates and $M_{12} \propto (V_{tb}V_{td}^*)^2$ with a negative coefficient of proportionality with our convention $CP|B^0\rangle = |\overline{B}^0\rangle$.

- It follows that $|M_{12}|/M_{12} = -e^{-2i\beta}$.

Combining all these results we find

$$\langle f|\mathcal{H}|B^0_{phys}(t)\rangle \propto e^{-\Gamma t/2} A_f \left[\cos\frac{\Delta m}{2}t + i\lambda_f \sin\frac{\Delta m}{2}t\right],$$

.

$$\langle f|\mathcal{H}|\overline{B}^0_{phys}(t)\rangle \propto e^{-\Gamma t/2} \overline{A}_f \left[\cos\frac{\Delta m}{2}t + i\frac{1}{\lambda_f} \sin\frac{\Delta m}{2}t\right],$$

where

$$A_f = \langle f|\mathcal{H}|B^0\rangle; \qquad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}^0\rangle,$$

and where

$$\begin{aligned} \lambda_f &= -\frac{|M_{12}|}{M_{12}}\frac{\overline{A}_f}{A_f} \\ &= \eta_f e^{-2i\beta} e^{-2i\phi_{wk}}. \end{aligned}$$

Observation:

ϕ_{wk} is the single weak phase in the amplitude for $B^0 \to f$. We see that $|\lambda| = 1$, a consequence of our assumptions that only one mechanism contributes to the decay and that $\Delta\Gamma$ can be ignored for B_d. The decay rate is then governed by

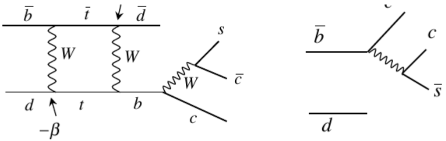
$$|\langle f|\mathcal{H}|B^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma t}\left[1 + \eta_f \sin 2(\beta + \phi_{wk}) \sin \Delta m t\right],$$

.

$$|\langle f|\mathcal{H}|\overline{B}^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma t}\left[1 - \eta_f \sin 2(\beta + \phi_{wk}) \sin \Delta m t\right].$$

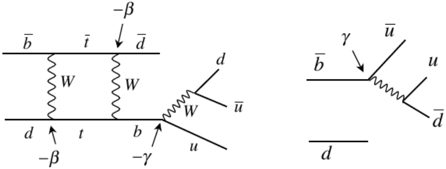
What is remarkable here is that there are no unknown matrix elements involving hadrons: when just a single weak phase occurs, the hadronic uncertainty disappears.

Examples of relevant decays



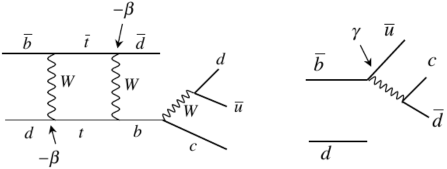
$B^0 \to J/\psi K_S$

B



$B^0 \to \pi\pi$

C



$B^0 \to D\pi$

Tagging

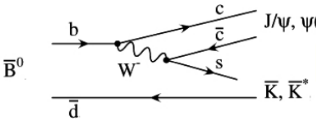
- Need to know how observed B began life.
- Observe other B and determine whether it is B^0 or $\overline{B}^0$.
- Determination will be imperfect.
- If it is wrong a fraction w of the time, $1 - A \sin \Delta mt$ becomes

$$(1-w)(1 - A \sin \Delta mt) + w(1 + A \sin \Delta mt) = 1 - DA \sin \Delta mt$$

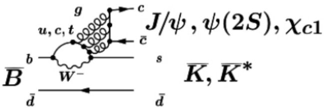
  where the dilution D is just $1 - 2w$.
- Figure of merit $Q = \sum \epsilon_i D_i^2$, where the ith tagging category captures a fraction ϵ_i of the neutral B events and has a dilution D_i.
- Most effective tagging method: charge of lepton from semileptonic decay
- But can also use charge of kaon or charge of secondary vertex

$\beta \equiv \arg\left[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*\right]$

$\sin 2\beta$ in golden $b \to \bar{c}cs$ modes



$V^*_{cb}V_{cs}$



◎ branching fraction: $O(10^{-3})$

the colour-suppressed tree dominates and the t penguin has the same weak phase of the tree

$$\lambda = \frac{q}{p}\frac{A(\bar{B} \to f)}{A(B \to f)} = \frac{V_{td}^* V_{tb}}{V_{td} V_{tb}^*}\frac{\bar{A}}{A} \sim e^{-i2\beta}\frac{\bar{A}}{A}$$

◉ $$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})}$$

S ~ sin2β

C ~ 0

◎ theoretical uncertainty:

◉ model-independent data-driven estimation from $J/\psi\pi^0$ data:

$$\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta = 0.000 \pm 0.012$$

◉ model-dependent estimates of the u- and c- penguin biases

$$\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta \sim O(10^{-3})$$

$$\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta \sim O(10^{-4})$$

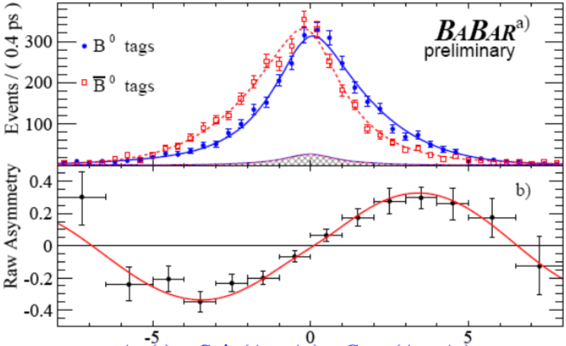
H.Li, S.Mishima
JHEP 0703:009 (2007)

H.Boos et al.
Phys. Rev. D73, 036006 (2006)

$\beta \equiv \arg\left[-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*}\right]$

$\sin 2\beta$ in golden $b \to \bar{c}cs$ modes

- The '*Golden Measurement*' of the B factories. The aims of this measurement were:
  - Measure an angle of the Unitarity Triangle.
  - Discover CP violation in B meson decays.



$$A_{CP}(t) = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)$$

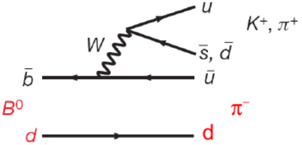
Sine term has a non-zero coefficient

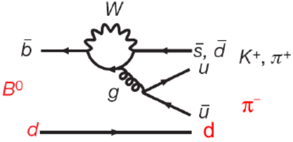
$S = \sin 2\beta = 0.671 \pm 0.024$

This tells us that there is CP violation in the interference between mixing and decay amplitudes in $\bar{c}cs$ decays.

Direct CP violation

- $B^0 \to K^{\pm}\pi^{\mp}$: Tree and gluonic penguin contributions

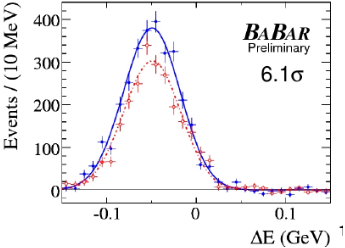




- Compute time integrated asymmetry

$$\mathcal{A}_{K^{\pm}\pi^{\mp}} \equiv \frac{N(\bar{B}^0 \to K^-\pi^+) - N(B^0 - K^+\pi^-)}{N(\bar{B}^0 \to K^-\pi^+) + N(B^0 \to K^+\pi^-)} = -0.098 \pm 0.012$$

  - Experimental results from Belle, BaBar, and CDF have significant weight in the world average of this CP violation parameter.
  - Direct CP violation present in B decays.
  - Unknown strong phase differences between amplitudes, means we can't use this to measure weak phases!



Putting it all together